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Galaxy Clustering: Why Peebles and Zel'dovich Were Both Right

For about two decades, two theories of galaxy formation based on gravitational clustering were popular. These were known as the "top-down" or "pancake" theory, and the "bottom-up" or "hierarchical clustering" theory. Hierarchical theories seem to fit much of the galaxy data, but this data also displays sheets and filaments long associated with the pancake theory. Recent numerical experiments have made it clear that the two theories are not mutually exclusive. In fact, hierarchical models display a surprising agreement with extrapolation of the long-wave part of the spectrum based on the Zel'dovich approximation, the basis of the pancake theory. Both approximations are limiting cases which are applicable with varying precision over a range of initial conditions. Both are needed to understand phenomena seen in N-body simulations. In particular the filaments seen in N-body simulations are "real," in the sense that they are a consequence of initial conditions, and can be detected even when there is very little power on large scales.

Key Words: *gravitational instability, cosmology, superclusters, galaxy clustering*

By about a quarter of a century ago, the first steps toward a theory of the clustering of matter in the universe and of experimental tests of theory had been taken.

Newton recognized that clumpiness would grow in a nearly uniform self-gravitating medium, but only much more recently were the linear solutions for perturbation growth in an expanding universe found.[1] During the 1950's and 1960's the Hot Big Bang model emerged as the favored cosmological theory and by the late 1960's

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specific and physically grounded work on structure formation began.

At that time, the main components of the early universe were assumed to be (baryonic) matter and radiation.[2] The modern obsession with nonbaryonic dark matter was far in the future, although hints had already appeared.[3-6] In such a medium, one can imagine two possible kinds of perturbations from homogeneity in the early hot plasma which could become galaxies later.

The first corresponds to perturbations in the baryon density, with constant radiation density. These used to be called "isothermal" perturbations, for obvious reasons. Today "isocurvature" is preferred; it has been noted that a more computationally tractable and likely scenario would have a modest radiation deficit accompanying a baryon excess so that the total enclosed mass is constant. Such perturbations cannot grow, because of the effect of radiation pressure, when the universe is still hot. But when the universe cools to a few thousand degrees, hydrogen atoms form and suddenly become nearly decoupled from radiation. Now the Jeans mass drops to about 10^6 $M_\odot$, and this is the characteristic scale of condensations one might expect to form first. It was noticed immediately[7] that this is close to the mass of a globular cluster. Since globular clusters have generally lower heavy element abundances than most galaxies and are presumed to be older, it is natural to speculate whether we are seeing the primordial units of structure.

Having done a bit of historical motivation, it is now convenient to sketch the basic tools used. The Fourier transform of the mass density $\rho(\mathbf{r})$ is

$$\delta_{\mathbf{k}} = \frac{\int \rho(\mathbf{r})\, e^{i\mathbf{k}\cdot\mathbf{r}}\, d^3r}{\langle\rho\rangle V_u} \tag{1}$$

where V_u is the integration volume. The power spectrum

$$P(k) = \langle\delta_k^2\rangle \tag{2}$$

where it is assumed we average over spherical shells of constant k. This quantity is especially useful for Gaussian distributions because it constitutes a statistically complete description of them. Gaussian distributions are especially interesting for cosmology because the Central Limit Theorem tells us they result from the sum of many uncorrelated random processes, and it is easy to imagine our ancestral density perturbations resulting from such. In fact, there is some evidence that Gaussian initial conditions are entirely sufficient to initiate clustering that would result in something like our present galaxy clustering pattern.[8] It is often assumed for the sake of simplicity that P is a power-law $P(k) \propto k^n$ where $-3 < n < 4$. The minimum of -3 is required to have convergence of density contrast on large scales; the maximum of 4 is required for stability on small scales and because if $n > 4$, then 4 will be generated anyway by dynamics.[9,10]

The density contrast δ_ρ/ρ is then

$$\left\langle \left(\frac{\delta_\rho}{\rho}\right)^2_x \right\rangle = \frac{V_u}{(2\pi)^3} \int d^3k \; P(k) \; W(kx) \tag{3}$$

where $W(kx)$ is a smoothing window to make the integral converge for nonzero P at large k. The density contrast is naturally a nondecreasing function of k.

The solutions for perturbation growth tell us that the contrast will grow steadily until it becomes nonlinear; at approximately this time it will decouple from the general expansion, and begin to collapse, presumably forming some sort of condensed object, such as a star or galaxy. Since δ is greater on small scales, they will collapse first; then those objects will merge to form larger ones. This is the hierarchical clustering picture. Since structures on larger scales are formed by the merging of small objects, it is natural to assume that the vagaries of location and mass of the small objects will determine their ultimate arrangement, so that it will be rather featureless. Therefore there has been no particular emphasis on interesting structures on very large scales as an expectation resulting from this point of view.

An in-depth treatment of the model is beyond the purview of this paper. The reader is referred to Peebles' excellent book[11] for greater depth, detail, and historical background. Solutions of the equations of motion are described there also.

The simplest thing to do (other than exact solutions with imposed symmetry) is to linearize the equations of motion. If the density is written as $\rho = \langle\rho\rangle(1 + \delta_\rho)$, application of the continuity equation, Euler equation, and Poisson equation gives

$$\delta(\mathbf{x}, t) = b(t)\delta(\mathbf{x}, t_i) \tag{4}$$

where b is a factor which can be calculated numerically[11]; but for a critical density universe b is just the expansion factor a between time t and t_i.

Another approach was suggested by Zel'dovich.[12] The Eulerian (co-moving) coordinates $\mathbf{r}$ of a matter point at time t are

$$\mathbf{r}(q, t) = \mathbf{q} + b(t)\nabla\varphi(q) \tag{5}$$

where $\mathbf{q}$ are the Lagrangian (i.e., initial, unperturbed) coordinates of the point. This is the famous "Zel'dovich approximation." Note that the motion (in co-moving coordinates) is essentially inertial; there is no change in velocity (with b as time parameter) and the *initial* velocity potential $\varphi(q)$ is used (Zel'dovich discusses the derivation of φ which is a simple multiple of the gravitational potential).

The classic first paper[12] is not clear on a number of points. Although he does not say so, his discussion of the solution indicates that the density field must be continuous and differentiable. It is clear that he knew of one cosmological scenario in which this would be true, the so-called "adiabatic" or "isoentropy" perturbations. Such perturbations assume that both baryons and photons are compressed, with their energy densities varying by the appropriate factor. Such perturbations oscillate before decoupling, but photon diffusion acts as a drag and damps them effectively up[13] to a scale of about 10^{12} $M_\odot$; therefore one would expect approximately uniform density patches with a diameter corresponding to this enclosed mass. Much later,[14] another scenario in which neutrinos provide the dark matter also provided a possibility for a smooth density field. The Zel'dovich approximation became associated with such simple damped density perturbation fields, which have P an increasing function of k up to some cutoff k_c, where it plunges rapidly toward zero.

Yet it is not obvious that this is all Zel'dovich had in mind. He does not exclude the "isothermal" theory as a valid place to apply his approximation. He realizes small scales will collapse first but asks ". . . should one apply the approximate solution to the 'gas' whose atoms are globular clusters or protostars or small gas clouds?" This comment seems to have been ignored for a long time. He posed the right question, but no one tried to answer it. The beginnings of an answer emerged quite by accident.

It is plain that the approximation (5) will move patches of finite size eventually into sheets or possible filaments as particles intersect each other. The first word used was "discs" but this is not quite right. At any rate, the earliest numerical simulations of pancake models[15] led to the realization that highly anisotropic structures formed. This was expected, but the interconnectedness (which led to topological analysis of superclusters)[16] was not (see Fig. 1). In retrospect, perhaps it should have been expected, but such comments are too easy to make. Still people were working within

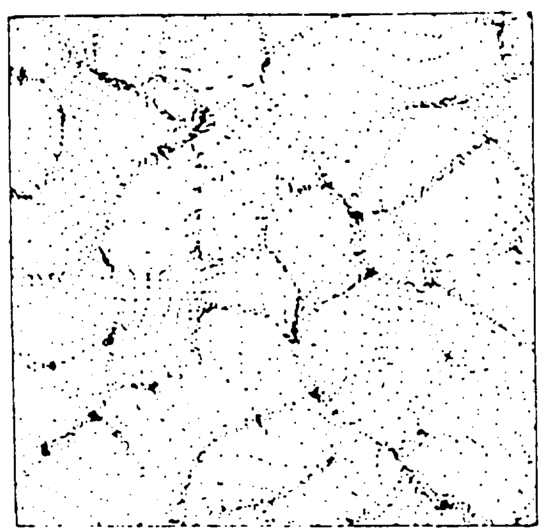

FIGURE 1 A figure drawn from the earliest simulation (Ref. 15) known to the author to have shown the interconnected cell structure arising from a truncated power spectrum. The simulation shown here is two-dimensional. Copyright Royal Astronomical Society (UK).

the framework of adiabatic perturbations or closely related phenomena. There is an excellent review of the development of this theory since 1970.[17]

The more basic question of whether the Zel'dovich approximation could be applied to hierarchical models appears to have been completely ignored even though it was clearly posed in his paper. The first hints appeared in the first simulation[18] of Cold Dark Matter (CDM) which does have nonzero power at large k. (It is not, however, a scale-free power-law spectrum.) This study used percolation techniques, which are a sensitive detector of filamentary structure, and got a result indicating that CDM produced an interconnected network much like that in the pancake model. This result was obtained independently in a different simulation[19] which used another approximation for the initial power spectrum, and a different computational method.

The reason for this filamentarity was not investigated for some time. There are two reasons for this: (a) There was a strong tendency in the field, which still exists, to regard simulations as tests of whether a given set of initial conditions can produce something that agrees with observational data, without asking why, without trying to unearth general principles. (b) Because the CDM power spectrum has a bend from $P(k) \propto k$ at small k to $P(k) \propto k^{-3}$ at large k, it appears that most of us simply assumed that the bend was sharp enough to behave like a cutoff, and that the model was acting like a pancake model due to this change in slope.

The issues were sharpened considerably by an unpublished letter sent by Peebles to about a dozen persons in the late 1980's. The letter argued that filaments should be broken up completely by the force generated by smaller condensations, and should not be seen. A number of possible numerical effects were suggested as possibly giving use to a false filament signal, including relics of the lattice on which many particles began, or the inevitable discreteness of the Fourier decomposition of the imposed power spectrum. This inspired a closer examination of possible numerical problems by relaxing some of these conditions,[20–22] successfully eliminating sources of false signals. No problems were found but the exercise proved interesting and stimulated further study.

In particular, an approach of controlled experiment was introduced,[21–23] in which phases of Fourier components δ_k were held constant for some or all of the range of wavenumbers in the initial perturbation spectrum. One could then compare, for example, pancake and hierarchical models in which the linear, long-wave conditions are the same. Thus, given the hypothesis that those long waves control the location of the filaments, these two simulations should have a family resemblance. They do.

In Fig. 2, we show one example of such a family. The initial spectra were characterized by $P(k) \propto k^n$ up to some cutoff k_c and $P = 0$ above that. The columns correspond to constant n and the rows to constant k_c. The family resemblance is strong, and clearly a function of n. This approach was used in our work.[21,22,24,25] A closely related and complementary independent approach[23] was developed in which families with identical power spectra and various random phases for $k > k_c$ were shown to strongly resemble each other. All of this established rather well by eye that hierarchical models which are going nonlinear on some wavenumber k_{NL} closely resemble pancake models which had the same initial conditions for $k < k_{NL}$ but have $P = 0$ initial conditions for $P < k_c = k_{NL}$, and that this resemblance is stronger for more negative n. It can still be detected by eye, for example, even for $n = 1$.

Detection by eye has been characterized as a "beauty contest," devoid of quantitative content. Correlation functions, stressed in modern cosmology by Peebles,[11] are a valuable way to make quantitative statements about structure. The cross correlation was recently introduced[22,24,25] to quantitatively compare different simulations. In this case, they had the same amplitudes and phases in their initial conditions for a range of small k, and different amplitudes for large k. To the extent that the distributions are determined by their long waves, they will be correlated. We adopted the definition

$$K_{ab} = \frac{\langle \delta_a \delta_b \rangle}{\sigma_a \sigma_b} \tag{6}$$

as our cross correlation coefficient. K is not a function of distance; we are assuming that δ's are measured at the same point, which implies some smoothing. We have also divided out the standard deviation of each density field, so that two identical distributions will have $K = 1$.

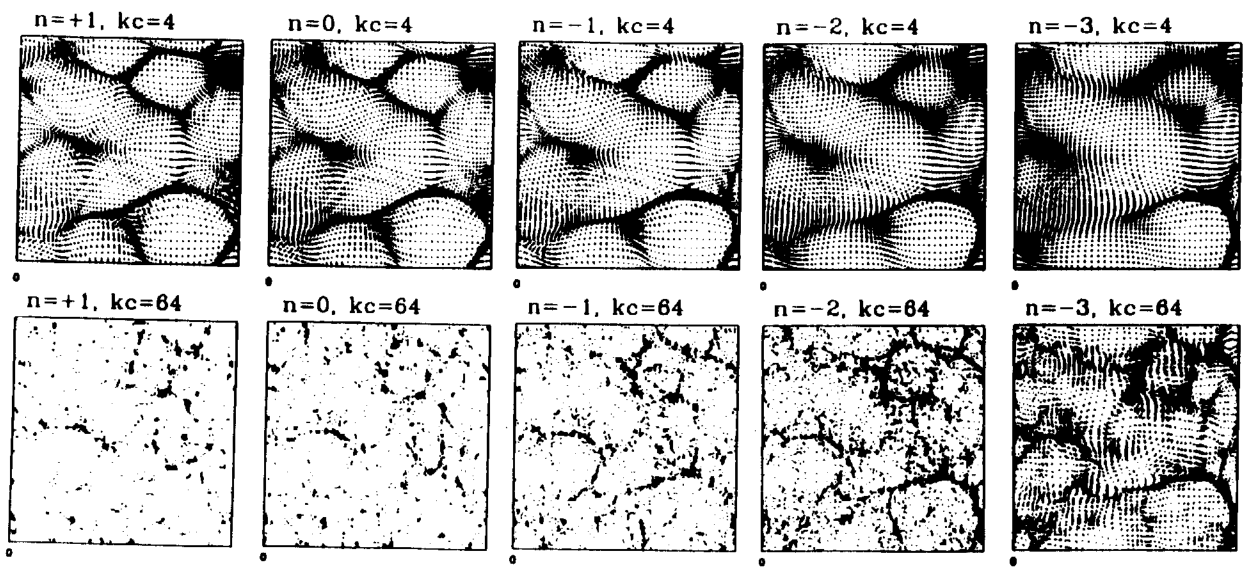

FIGURE 2 The positions of a subset of the particles in thin slices of a family of three-dimensional simulations with identical phases for all δ_k. The initial spectra have the specified power-law index n and cutoff k_c (in units of the fundamental model of the box). All are just going nonlinear at $k = 4$. Copyright Astrophysical Journal, 1993.

Applying this to the simulations verified the beauty contest conclusion: there is a definite correlation between pancake and hierarchical models with the same phases. This correlation is weaker for more positive n but is still significantly different from zero for $n = 1$.

The existence of filamentary structure in redshift surveys, once dismissed as the result of overactive imagination, is now incontrovertible. However, we now understand clearly that this does not imply that the pancake model correctly describes galaxy formation. In order to get a reasonable density contrast, it seems necessary to use a hierarchical model and assume that galaxies form slightly more efficiently in high density regions (called biasing).[8] On the other hand, it is clear that pancake dynamics are generic in gravitational clustering, and describe the construction of superclusters. We can now answer "yes" to the widely ignored question posed by Zel'dovich in 1970.

Ten years ago, pancake and hierarchy were rival theories of clustering, regarded as mutually exclusive. We now know that they are valuable and complementary descriptions of a universal process.

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